Problem 1. Answer: 6. The numbers of votes are distinct. They are 10, 9, 8, 7, 6, in order for the minimum to be as large as possible.

Problem 2. Answer: 1. Only 555555555 has its digits summing to a multiple of 9 which is necessary and sufficient for a number to be divisible by 9.

Problem 3. Answer DBCA. D<1

Problem 4. Answer: 5. 9 slices have 18=4·4+2 sides, thus require at least 5 minutes. One way to toast all 18 sides in 5 minutes: toast 1, 2, 3, 4, then 3, 4, 5, 6, then 1, 2, 5, 6, then 7, 8, 9, then 7, 8, 9.

Problem 5. Answer: (2.10, 2.30). Let m be the cost of the main course and d be that of the dessert. Solving m−d=22.5, m+d=12d, we get d=2.25.

Problem 6. Answer: 29. 8^n=2^{3n} will divide 4444=2^{88}·11^{44} if and only if 3n<=88.
The largest such n is 29.

Problem 7. Answer: 7/8. A=7/8=1·1/8, B=6/7=1·1/7, C=5/6=1·1/6, D=4/5=1·1/5, and E=3/4=1·1/4.

Problem 8. Answer: CD. A, B and E must sit on the same type of seat, which must then be stools. So C and D sit on chairs.

Problem 9. Answer: 144 days. (12·60)/5=144 days.

Problem 10. Answer: 9. x^2+a=6x has a unique solution if and only if 6·6-4a=0. So a=9.

Problem 11. Answer: (60, 75). Focus on one roller, say the left one. Imagine a mark on the safe above the roller, and another mark on the ground below the roller. After one revolution of the roller, the mark on the safe is 2·pi·5=10·pi inches to the right of the contact point of the roller and the mark on the ground is an equal distance to the left of the contact point.
The safe has moved 20pi inches, and 60<20pi< 75.


Problem 13. Answer: 2. (0+a)^3+b=1 and (1+a)^3+b=2 are easily solved by elimination to get the two solutions (a,b)=(0,1), (-1,2).

Problem 14. Answer: 4.5. Let the hill be d miles in length.
Then it takes d/4 +0.5d/6 hours for the 1.5d miles trip. The average speed, the ratio between the distance traveled and time, is 4.5 mph.

Problem 15. Answer: 31-45.
The exterior angle in degrees of a regular pentagon is 360/5=72. This implies (i) the interior angle is 108, and (ii) angle ACB=angle ECD=72/2=36.
Combining, angle ACE=108-36-36=36.

Problem 16. Answer: 250

Problem 17. Answer: 3. There cannot be an odd number of dumbbells that weigh 30 lbs, else the total weight would be an odd multiple of 10.

Problem 18. Answer: 2781 and 2900. For the sum to be 27 the first two digits must be at least 9. Observe that 1999 doesn't work, so the first digit must be (at least) a 2. So 2799 is the first year.

Problem 19. Answer: 11. The k-th cut will add at most k more pieces. This is feasible by always making new cuts intersect all previous cuts in the region. So 1+1+2+3+4=11.

Problem 20. Answer: 4. Squaring this positive number x gives x^2=7+4·3^{1/2}+2\sqrt{1+7·4·3^{1/2}}=16.
Thus, x=4.
Problem 21. Answer: 500050. The last elements in the 99th and 100th row are
99·100/2=4950 and 100·101/2=5050, respectively. The sum of the
100 numbers in the 100th row then is (4951+5050)-100/2=500050.

Problem 22. Answer: 2.5. $s_1>s_2$. Let the tail end of the train be
distance $d$ away from the runner and $r=s_1/s_2>1$.
Then $t_1=d/(s_1-s_2)$,
$t_2=d/(s_1)$,
$r=t_2/(s_1+s_2)/(s_1-s_2)=1/(1-r)$. Thus, $r=1+2^{1/2}$ which is approximately 2.41.

Problem 23. Answer: 23. $p(x+1)-p(x)$ must be linear,
and $p(x+1)-p(x)$ has values of 2 and 3 at $x=1$ and 2.
Thus, $p(5)=p(0)+2+3+4+5+6=23$.

Problem 24. Answer: 0.5°. The speed of the minute hand is 6° per minute.
That of the hour hand is 1/12 as much, that is, 0.5°. The smallest angle occurs
when the time is closest to when the two hands meet. The meeting
instances are $n$ o’clock $60n/11$ minutes, $1\leq n\leq10$.
When $n=2$ or 9, $60n/11$ is 1/11 minute away from an integer minute
which is the smallest possible. The angle at that time is $(6-0.5)\cdot1/11=0.5°$.

Problem 25. Answer: 1.9. Let the length of one edge be $a$.
Then the height of a regular triangle is $3^{1/2}a/2$ and
that of the tetrahedron is $6^{1/2}a/3$.
The unit radius $r$ satisfies $1=r=(3/4)\cdot6^{1/2}a/3$,
so $a=2\cdot6^{1/2}/3$. 