

# 1 Nobody Understands Quantum Mechanics

## 1.1 Quantum Blues

Richard Feynman, who won the Nobel prize in physics in 1965, is often quoted as saying that nobody understands quantum mechanics:<sup>1</sup>

There was a time when the newspapers said that only twelve men understood the theory of relativity. I do not believe there ever was such a time. There might have been a time when only one man did, because he was the only guy who caught on, before he wrote his paper. But after people read the paper, a lot of people understood the theory of relativity in some way or other, certainly more than twelve. On the other hand, I think I can safely say that nobody understands quantum mechanics. . . . I am going to tell you what nature behaves like. If you will simply admit that maybe she does behave like this, you will find her a delightful entrancing thing. Do not keep saying to yourself, if you can possibly avoid it, 'But how can it be like that?' because you will get 'down the drain,' into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that.

What is the theory of relativity about? Even non-physicists are likely to say that it's about space and time. The notion of space as a sort of universal three-dimensional arena in which events take place, and time as the ticking of a universal clock, turns out to be wrong. According to Einstein's theory of special relativity,<sup>2</sup> space and time depend on the state of motion of a system and so are different for Alice on a flight to Rome and Bob on a train to New York, and quite a lot different for neutrinos moving at light speed. Hermann Minkowski, Einstein's former mathematics professor, showed that space and time in special relativity can be represented by a four-dimensional non-Euclidean geometry and predicted that 'space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.'<sup>3</sup> General relativity goes further and treats gravity as the bending of space-time, so objects falling under the influence of gravity move by following the curvature of space-time.

One might think that the idea of space and time fading away into 'mere shadows' relative to some sort of merging of the two notions is pretty wild and not that easy to get one's head around. How is it that 'nobody understands quantum mechanics,' but there isn't a similar difficulty in making sense of the theory of relativity? There's an introduction to special relativity in the *More* section at the end of this chapter. It's not hard to see how Alice and Bob, equipped with identical synchronized

clocks, could disagree about how much time passes between two events, or about the distance between them, given the two central assumptions of the theory.

If relativity is about the geometrical structure of space-time, what is quantum mechanics about? There are a surprising variety of answers to this question: that quantum mechanics is about energy being quantized in discrete lumps or quanta, or about particles being wavelike, or about the universe continually splitting into countless co-existing quasi-classical universes, with many copies of ourselves, and so on. A rather more mundane answer, with quite remarkable implications, has emerged over the past thirty years or so from the study of the difference between classical information and quantum information: quantum mechanics is about new sorts of probabilistic correlations in nature, so *about the structure of information*, insofar as a theory of information in the sense relevant to physics is essentially a theory of probabilistic correlations.

Here's a very short history of the birth of quantum mechanics. A hot solid body glows, which means that it emits light. If you pass the light through a prism, you see a continuous band of colors like the spectrum of a rainbow, corresponding to light waves of varying frequencies. For a hot gas, what you see is an 'emission spectrum' of discrete colored lines with dark gaps between the different colors. A cool gas surrounding a hotter object like a star produces the opposite effect: a continuous 'absorption spectrum' with dark lines at the discrete frequencies of the emission spectrum of the gas. Different gases have different spectra, with the lines in different places. These facts were known in the nineteenth century. The problem was that no one could figure out how to explain the discrete spectra in the conceptual framework of classical physics.

In 1913, the Danish physicist Niels Bohr proposed a new theory of atomic structure<sup>4</sup> as an explanation. In Bohr's theory, negatively charged electrons orbit a positively charged nucleus, like planets orbiting a star. There's a difference between the behavior of an orbiting planet, described by Newton's theory of gravity, and the way a charged particle moves in an electromagnetic field, described by Maxwell's theory of classical electrodynamics. The details are irrelevant to what follows, but the significant difference here is that, according to classical electrodynamics, a charged particle emits electromagnetic radiation when it accelerates and so loses energy, unlike an accelerating Newtonian body. An electron orbiting the nucleus of an atom is constantly accelerating (because its velocity is constantly changing direction), and so should radiate energy in the form of light and spiral into the nucleus as it loses energy. That doesn't happen, so Bohr's theory stipulates that the energy of an orbiting electron is 'quantized'—there's a discrete set of allowed orbits associated with different energies that an electron can occupy, with lower energy orbits closer to the nucleus. The theory also stipulates that an atom radiates or absorbs energy only when an electron jumps from one of these quantized orbits to another orbit, with the frequency of the radiation depending on the energy gap between the two orbits. An emission spectrum is a picture of the radiation emitted when electrons jump from higher to lower energy orbits in the gas atoms. An absorption spectrum represents electron jumps from lower to higher energy orbits, when the atoms absorb energy radiated by a hotter body.

Bohr's theory explains the distribution of spectral lines for a gas in terms of its atomic structure, but the ad hoc 'quantum' rules for orbiting electrons conflict with classical electrodynamics, and also with classical mechanics, Newton's theory of motion. In 1925, Heisenberg published a breakthrough paper<sup>5</sup> in the journal *Zeitschrift für Physik* and shortly afterwards developed the idea into

an early version of quantum mechanics in collaboration with Max Born and Pascual Jordan. The title of Heisenberg's paper in English is 'On the quantum-theoretical re-interpretation [Umdeutung] of kinematical and mechanical relations.' The 'Umdeutung' changes the ball game. The thought was that the discrete orbits were an artificial theoretical fix that 'saved the appearances,' but this was not the right way to think about the structural features of atoms responsible for the spectral lines. Heisenberg proposed to *re-interpret* classical mechanical quantities, like position, momentum, energy, angular momentum, as operations, later represented by operators that act on and transform the states of quantum systems. The aim was to explain the discrete frequencies of light emitted by atoms *without appealing to electron orbits*, and later to explain other phenomena that couldn't be explained by classical physics. In a 1925 letter to Wolfgang Pauli, Heisenberg wrote:<sup>7</sup>

All of my meagre efforts go toward killing off and suitably replacing the concept of the orbital paths that one cannot observe.

In Heisenberg's theory, the effect of applying an operation  $A$  followed by an operation  $B$  can differ from applying  $B$  followed by  $A$  for certain operations, which is to say that quantities  $A, B$  represented by operators needn't commute with each other: you can have  $AB \neq BA$  in terms of the effect on a quantum state. If that's so, then it turns out that quantum systems can't have definite values for all these quantities simultaneously—in particular, an electron can't have definite position and momentum values and so can't have a well-defined orbit in an atom. If a quantity has a definite value in a quantum state, then certain other quantities are indefinite, and what you find if you measure a quantity with an indefinite value is intrinsically random, in the sense that the outcome is independent of any information available before the measurement, as I'll show in Chapter 4, *Really Random*.

Indefiniteness or intrinsic randomness is related to a feature of the theory that Heisenberg later formulated as an 'uncertainty' or 'indeterminacy' principle, but it's a much more radical departure from classical or commonsense ways of thinking about physical systems than the uncertainty principle. As derived in quantum mechanics, the uncertainty principle is simply a statement about a reciprocal relation between two noncommuting quantities, or 'observables' in the jargon of quantum mechanics. Quantum states assign probabilities to measurement outcomes. If an observable is most likely to have a value in a certain range of values when measured, the likely outcomes of measuring a noncommuting observable can't be pinned down more precisely than a range of values that is reciprocally related to the first range: when one range of values is small, the other is correspondingly large. In particular, the uncertainty principle says that it's impossible to prepare a system in a quantum state in which two noncommuting observables both have definite values, so the observables are said to be 'incompatible.'

The uncertainty principle is open to various interpretations, and Heisenberg himself explained the relationship in the 1927 paper in which he introduced the principle as the result of irreducible measurement disturbances. He argued that the procedure for measuring an observable necessarily disturbs the value of an incompatible or noncommuting observable—which presupposes that there are definite values there to be measured or disturbed in the first place. An uncertainty principle in this sense would be a feature of any theory with noncommuting observables, but the indefiniteness

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of observables, and the intrinsic randomness of the value revealed when an observable of a quantum system is measured, is a feature of the particular way in which commuting and noncommuting observables are related in quantum mechanics.

Schrödinger published a wave mechanical version of the theory in 1926<sup>8</sup> that kept the orbits and explained their quantization as a wave phenomenon. Shortly afterwards, he proved the formal equivalence of Heisenberg's noncommutative mechanics and his own wave mechanics. Not surprisingly, physicists found wave mechanics more intuitively appealing as a picture of reality at the subatomic level than the abstract notion of a noncommutative mechanics, but the intuitive appeal is misleading. As Schrödinger pointed out in a lecture to the Royal Institution in London in March, 1928, the wave associated with a quantum system evolves in an abstract, multi-dimensional representation space, not real physical space:<sup>9</sup>

The statement that what *really* happens is correctly described by describing a wave-motion does not necessarily mean exactly the same thing as: what *really* exists is a wave-motion. We shall see later on that in generalizing to an *arbitrary* mechanical system we are led to describe what really happens in such a system by a wave-motion in the generalized space of its co-ordinates (*q*-space). Though the latter has quite a definite physical meaning, it cannot very well be said to 'exist'; hence a wave-motion in this space cannot be said to 'exist' in the ordinary sense of the word either. It is merely an adequate mathematical description of what happens. It may be that also in the case of a single mass-point, with which we are now dealing, the wave-motion must not be taken to 'exist' in *too* literal a sense, although the configuration space happens to coincide with ordinary space in this particular simple case.

That's the end of the short history. The point is not to suggest that 'Heisenberg saw it all in 1925,' or that Heisenberg had anything like an information-theoretic interpretation in mind. It's rather that the 'Umdeutung' contained the germ of a radically new way of thinking about physical systems that developed as Heisenberg's idea was applied to more complex systems, while Schrödinger's wave mechanics evoked a very different structural picture that has turned out to be misleading in many ways and the source of a lot of confused thinking—not necessarily Schrödinger's fault.

The idea of a wave as a representation of quantum reality continues to shape contemporary discussions of conceptual issues in the foundations of quantum mechanics. But, as Schrödinger pointed out, it is 'merely an adequate mathematical description of what happens.' From the perspective adopted here, the later formalization of quantum mechanics by Paul Dirac<sup>10</sup> in 1930 and John von Neumann<sup>11</sup> in 1932 as a theory of observables represented by operators on a space of quantum states is fundamentally an elaboration of Heisenberg's 'Umdeutung' rather than a wave theory. Operators needn't commute—the order in which they are applied generally makes a difference—and the really significant thing about a noncommutative mechanics is the novel possibility of correlated events that are *intrinsically random*, not merely apparently random like coin tosses, where the outcome of a toss is determined by the way a coin is tossed, and the probabilities of 'heads' and 'tails' represent an averaging over differences among individual coin tosses that we don't keep track of for practical reasons. This intrinsic randomness allows *new sorts of nonlocal*

*probabilistic correlations* for ‘entangled’ quantum states of separated systems, where the probabilities are, as von Neumann put it, ‘perfectly new and *sui generis* properties of physical reality.’<sup>12</sup> Schrödinger, who coined the term, referred to entanglement (‘Verschränkung’ in German) as ‘the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought.’<sup>13</sup> So the deep significance of Heisenberg’s ‘Umdeutung’ is that quantum mechanics, as a noncommutative modification of classical mechanics, is a theory about a *structurally different sort of information* than classical information.

Quantum entanglement plays a major role throughout the book. At this point I haven’t said what it is, other than that entanglement is somehow associated with ‘new sorts of nonlocal probabilistic correlations.’ I’ll say more about entanglement, and about quantum states and operators, when I talk about polarization in Chapter 2, *Qubits*.

What do correlations have to do with information? The classical theory of information was initially developed by Claude Shannon to deal with certain problems in the communication of messages as electromagnetic signals along a channel such as a telephone wire. A communication set-up involves a transmitter or source of information, a communication channel, and a receiver. An information source produces messages composed of sequences of symbols from an alphabet, with certain probabilities for the different symbols. The fundamental question for Shannon was how to quantify the minimal physical resources required to represent messages produced by a source, so that they could be communicated via a channel and reconstructed by a receiver. For this problem, and related communication problems, the meaning of the message is irrelevant.

As Shannon remarked in his seminal paper ‘A Mathematical Theory of Communication’:<sup>14</sup>

The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have *meaning*; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one *selected from a set* of possible messages. The system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design.

So a theory of information for Shannon is about the ‘engineering problem’ of communicating messages over a channel efficiently. In this sense, the concept of information has nothing to do with anyone’s knowledge and everything to do with the stochastic or probabilistic process that generates the messages. The standard unit of information is the bit, short for ‘binary digit.’ An information source that produces sequences of 0’s and 1’s from a two-symbol alphabet, with equal probability for each symbol, is said to produce one bit of information per symbol. Shannon showed that it’s possible to compress the information required to communicate a message—to reduce the average number of bits per symbol—up to a certain optimal compression, if the probabilities of the different symbols produced by an information source are not all equal.

In modern formulations, information theory is about random variables—variables that take values from a set of values with certain probabilities—and correlations between random variables. As such, information theory is a branch of the mathematical theory of probability, and the physical

communication of messages is just one application. In proposing that quantum mechanics is about the structure of information, I mean that the theory deals with new sorts of probabilistic correlations that are structurally different from correlations that arise in classical theories (and, of course, the theory is also able to handle standard classical correlations). What we have discovered is that the ‘engineering’ possibilities for representing, manipulating, and communicating information in a quantum world are different than we thought, irrespective of what the information is about.

John Wheeler’s slogan ‘it from bit’<sup>15</sup> and Rolf Landauer’s influential comment that ‘information is physical’<sup>16</sup> are often taken to suggest that information is primary and that ‘stuff’—what physics is usually understood to be about—is in some sense derived from information. Vlatko Vedral, for example, in his book *Decoding Reality* explicitly endorses the view that ‘our reality is ultimately made up of information’ and that ‘the laws of Nature are information about information.’<sup>17</sup>

The idea that the basic building blocks of reality might be information is dizzyingly intriguing, but I have no idea how to make sense of this. It can’t be information in Shannon’s sense. To say that quantum mechanics is about the structure of information is not to say that ‘stuff’—particles, fields, planets, people—is somehow made of information. The theory of relativity rests on the recognition that events have a spatio-temporal structure, and that this structure is not what Newton thought it was. This is not usually taken to mean that ‘stuff’ is made of space and time (although there have been such suggestions, notably by Wheeler). Similarly, the conceptual revolution in the transition from classical to quantum physics should be understood as resting on the recognition that there is an *information-theoretic structure* to the mosaic of events, and this structure is not what Shannon thought it was.<sup>18</sup>

Of course, this is not a description of the actual historical development of quantum mechanics—Shannon published ‘A Mathematical Theory of Communication’ in 1948 and Heisenberg’s ‘Umdeutung’ paper appeared in 1925. Rather, this book develops the idea that what is revolutionary about quantum mechanics is analogous to what is revolutionary about the theory of relativity: a fundamental structural change in the way we represent how events fit together, where the change involves spatio-temporal structure in the case of relativity, and the structure of information in the case of quantum mechanics.

The claim that quantum mechanics is about the structure of information is often met with John Bell’s dismissive response: ‘*Information? Whose information? Information about what?*’<sup>19</sup> But we don’t ask these questions about a USB flash drive. A 64 GB drive is an information storage device with a certain capacity, and whose information or information about what is irrelevant.

 **The bottom line**

- Quantum mechanics is fundamentally a theory about the structure of information, insofar as a theory of information in the physical sense is essentially a theory of probabilistic correlations.
- This is largely implicit in Heisenberg's 'Umdeutung' paper, in which classical quantities like position and momentum are 're-interpreted' as operations, later represented by operators that act on and transform the states of quantum systems. Operations needn't commute—the order in which they are applied to a quantum state can make a difference—and a noncommutative mechanics allows the novel possibility of intrinsically random events associated with new sorts of nonlocal probabilistic correlations for so-called 'entangled' quantum states of separated systems.

## 1.2 Why Bananaworld?

What is it about quantum correlations that leads prominent physicists like Feynman to say that 'nobody understands quantum mechanics'? The usual way of approaching this question is from the familiar perspective of classical physics, or from commonsense intuitions about correlations, but that's a limited perspective that risks introducing implicit assumptions and prejudices. The idea of Bananaworld is to look at quantum correlations 'from the outside.' Bananaworld is an imaginary world in which there are classical and quantum correlations, but also superquantum correlations between separated systems that are even more nonclassical than the correlations of entangled quantum states. The conceptual puzzles of quantum correlations arise without the distraction of the mathematical formalism of quantum mechanics, and you can see what is at stake—where the clash lies with the usual presuppositions about the physical world.

In the contemporary quantum information literature, you'll find references to 'Boxworld.'<sup>20</sup> A 'box' is an imaginary device, with an input and output port on one side for Alice and an input and output port on the other side for Bob. Inputs can be 0 or 1, and outputs can be 0 or 1 (and, more generally, there could be more than two inputs, or more than two outputs). A box is defined by a particular correlation between inputs and outputs. So a box is really just an abstract device that produces a correlation, classical, quantum, or superquantum, without any specification of an internal mechanism that could produce the correlation. A nonlocal box can be stretched by an arbitrary amount, or perhaps separated into two parts, an Alice part and a Bob part, by any distance, without affecting the correlation. A box can be used only once: all it does is produce an output following an input at each side (whether or not there is an input at the other side), and once that happens the box is done for that side—you need a new box for a new input. Boxworld has turned out to be a powerful conceptual tool in exploring nonclassical features of quantum mechanics. In effect, Bananaworld is Boxworld, with bananas and peelings and tastes instead of boxes with 0, 1 inputs and outputs.

The protagonists of information-theoretic scenarios are Alice and Bob (and sometimes Charlie or Clio as well), who are in different locations and can each choose to perform one of at least two

alternative actions—say Alice can choose to measure one of two noncommuting or incompatible observables labeled  $A$  and  $A'$  on a quantum system in her possession, and Bob can choose to measure  $B$  or  $B'$  on his system—with at least two possible outcomes in each case, say 0 or 1. Here  $A$  and  $A'$  are just labels for Alice's choices, and  $B$  and  $B'$  are labels for Bob's choices, where  $A$  and  $B$  might represent the same measurement choice by Alice and by Bob, but could also represent different choices (and similarly for  $A'$  and  $B'$ ). Once Alice or Bob measures a particular observable, the quantum system is changed in an intrinsically random way and is no longer in the same quantum state, so no longer available in the original state for the alternative choice. The fact that a choice is free (in a precise sense clarified in Chapter 4, *Really Random*) turns out to be essential here. So there must be at least two actions, with at least two possible outcomes for each action, because otherwise there would be no variation and so no correlation. The choice of action could be random, so Alice and Bob could be replaced by random number generators that produce 0's and 1's with equal probability, corresponding to the two possible choices, 0 for  $A$  and 1 for  $A'$  for Alice, and 0 for  $B$  and 1 for  $B'$  for Bob.

Correlations are correlations, irrespective of the nature of the systems that manifest the correlations, and the discussion needn't be confined to measurement correlations. So imagine discovering Bananaworld, an island covered with banana trees. A banana tree is really a giant herb, and bananas grow in bunches pointing up from the stem, so the non-stem end is actually the top end. There are two, and only two, ways to peel one of these bananas (two possible actions that can be performed). Some primates on the island peel a banana from the stem end ( $S$ ), while other primates prefer peeling from the top end ( $T$ ). A Bananaworld banana simply can't be peeled any other way—suppose, if you like, that a banana is simply inedible if you try to get at the fruit without peeling from the stem end or the top end. Once a banana is peeled a certain way, of course, it's a peeled banana—the alternative peeling is no longer available—and it tastes just like an ordinary banana ('o' or 0), or the flavor is intense, incredible, indescribably delicious ('i' or 1). Whether the taste is 0 or 1 is an objective fact, not a subjective matter of opinion, and anyone who checks will agree that the taste is definitely ordinary or definitely intense. Various correlations between the tastes (0 or 1) of bananas and alternative peelings ( $S$  or  $T$ ), which persist when bananas from the same bunch are separated by any distance, can be considered, with features that correspond to classical, quantum, or even superquantum correlations. The same conceptual problems that arise for quantum systems like electrons or photons arise for bananas in Bananaworld.

If you're not used to thinking about Boxworld, it's less confusing to talk about peeling a banana in one of two ways,  $S$  or  $T$ , with two possible tastes, ordinary (0) or intense (1) in each case, than to talk about two inputs, 0 or 1, to a box, with two possible outputs, also represented by 0 and 1, for each input. To compare with measurement correlations: a particular peeling,  $S$  or  $T$ , by Alice or Bob corresponds to the measurement of a particular quantum observable,  $A$  or  $A'$  for Alice, or  $B$  or  $B'$  for Bob, and the two possible tastes correspond to the two possible measurement outcomes. Alice's two observables are noncommuting or incompatible, as are Bob's, so measuring one observable precludes measuring the other, just as peeling  $S$  or  $T$  precludes the alternative peeling. I'll say more about the observables in question as I go along (in the *Bananaworld* chapter, peeling  $S$  or  $T$  corresponds to measuring the polarization of a photon in one of two directions  $45^\circ$  apart, or measuring the spin of an electron in one of two orthogonal directions), but all that



Figure 1.1: Alice and Bob peel two bananas from the same two-banana bunch in Bananaworld, where tastes and peelings are correlated in a particular way. Alice peels her banana from the top end ( $T$ ) and Bob peels his banana from the stem end ( $S$ ). In this case, both bananas taste the same: *incredible*, *intense*, *indescribably delicious* (1). The correlation could be an instance of the Popescu–Rohrlich correlation described in Chapter 3, *Bananaworld*.

matters here is that the two ways of peeling a banana correspond to measuring two noncommuting or incompatible quantum observables.

The idea is to get at what’s puzzling about quantum correlations by considering the extent to which Alice and Bob, limited to certain resources, can simulate various correlations in Bananaworld. A simulation can be thought of as a game between Alice and Bob, and a moderator, played over several rounds. Think of a simulation game as like a TV game show. At the beginning of each round of a simulation game, the moderator, who can communicate with Alice and Bob separately, gives each player one of two prompts,  $S$  or  $T$ , and the player is supposed to respond to his or her prompt with one of two responses, 0 or 1. They win a round if the responses and the prompts are correlated in the right way. Alice and Bob are allowed to discuss strategy before the first round, and

they are then sent to separate locations (say, separate soundproof booths from which neither player is visible to the other, each separately linked by telephone to the moderator) and are not allowed to communicate with each other during the game. The game is played over many rounds, and at the end of the game they win a prize, where the value of the prize depends on the number of rounds they win. So the aim is to figure out an optimal strategy: a strategy that will enable them to win the maximum number of rounds.

Of course, Alice and Bob could win a round of the game, or several consecutive rounds, purely by chance even if they respond randomly without any strategy. The relevant question is whether there is a winning strategy for the simulation game, assuming Alice and Bob have access to certain resources. For example, Alice and Bob might be allowed to take pencil and paper or calculators with them to perform calculations when they are separated, or written instructions for responding to a given prompt that they prepare during the strategy session before they are separated. Or they might be allowed to base their responses on the outcomes of measurements on shared pairs of entangled quantum particles they prepare during the strategy session, and so on.

Some correlations can be perfectly simulated with local resources available to Alice and Bob separately. These could be local lists of instructions that tell Alice and Bob separately how to respond to prompts ('local' in the sense of a list of instructions for Alice independent of Bob's instructions, and a list of instructions for Bob independent of Alice's instructions). Other correlations can't be simulated with local resources, but can be perfectly simulated with shared nonlocal entangled quantum states. Still other correlations can't be perfectly simulated with quantum resources, but a simulation with shared entangled quantum states does a better job than the optimal simulation with local resources. So the plan will be to set things up in terms of a correlation in Bananaworld that is counterintuitive in some way, and the punchline will then be to show that the correlation can be simulated with nonlocal quantum resources but not with local resources, or that a quantum simulation is better than the best possible simulation restricted to local resources. If Alice and Bob can't simulate a correlation with local resources, but can succeed or do better with entangled quantum states, *that's a fact about our world*, not just about Bananaworld. The point is to use Bananaworld in this way to say something significant about our quantum world.

Local quantum resources provide no advantage over local classical resources, so I'll sometimes refer to correlations that can be perfectly simulated with local resources as classical correlations. For classical correlations, there is a winning strategy for the simulation game that does not involve access to a nonlocal resource like entangled quantum states and measuring instruments capable of measuring quantum observables. Specifically, local lists of instructions that could include shared lists of random numbers generated before the start of the game ('shared randomness') are allowed as classical resources, but not shared copies of a pair of particles in an entangled quantum state and appropriate quantum measuring instruments, the quantum analogue of classical shared randomness. If there is no winning strategy, the interesting question is whether there is an optimal strategy, and what that would be. I'll give a precise characterization of the difference between classical and quantum correlations in Chapter 3, *Bananaworld*.

Correlations between events at different places cry out for explanation, as John Bell put it,<sup>21</sup> and in classical physics or, for that matter, in everyday life, there are just two sorts of explanation. Either there is a direct causal connection between the events, a physical signal that takes a certain

amount of time to travel continuously from one event to the other and transmit information between them, or there is a common cause: some event in the common past of the correlated events that is responsible for the correlation, like a flash of lightning that is the common cause of Alice and Bob both hearing thunder at more or less the same time if they are located at different places in the vicinity of a thunderstorm. Nothing moves between Alice and Bob. Rather, a sound wave, the common cause of the vibration in their eardrums, moves from the disturbance in the atmosphere to Alice and Bob.

In Bananaworld there are correlations between the tastes of bananas from the same bunch peeled in various ways by Alice and Bob, even if they peel their bananas in separate locations on the island, where *both types of explanation are excluded*. As I'll show in the *Bananaworld* chapter, bananas correlated in this way can't have definite or pre-determined tastes before they are peeled, and in Chapter 4, *Really Random*, I'll show that in this case a banana tasting 0 or 1 when it's peeled is an intrinsically random event. Feynman's comment that nobody understands quantum mechanics applies with equal force to Bananaworld, but here it's clear that the mystery has to do with the weirdness of the nonclassical correlations rather than the properties of microsystems. The bananas have ordinary properties in the objective sense—what's extraordinary and counterintuitive is the nature of the correlations between peelings and tastes.

#### The bottom line

- Correlations can be defined for two possible actions performed by Alice and by Bob, with two possible outcomes for each action. Bananaworld makes this strategy concrete. The two possible actions correspond to two ways of peeling a banana, from the stem end (*S*) or the top end (*T*), and the two possible outcomes correspond to two possible tastes, ordinary (0) and intense (1). Specific correlations are considered in Chapter 3, *Bananaworld*, and subsequent chapters.
- Correlations between events at different places cry out for explanation. In classical physics, there are just two sorts of explanation: either there is a direct causal connection between the events, or there is a common cause: some event in the common past of the correlated events that is responsible for the correlation.
- In Bananaworld there are correlations between the tastes of bananas from the same bunch peeled in various ways by Alice and Bob, even if they peel their bananas in separate locations on the island, where *both types of explanation are excluded*.

### 1.3 Yes! We Have No Bananas,<sup>22</sup> But ...

Bananaworld is a possible world. You could imagine discovering an island like Bananaworld, but of course there are no banana trees in our world like the superquantum banana trees in Bananaworld. The really amazing thing about our world is that there are correlations that are closer to these Bananaworld correlations than classical correlations. What's mind-boggling is the discovery that

we live in a world in which there are nonlocal correlations, where a direct causal influence can be excluded as an explanation, that can't have a common cause explanation either.

As in Bananaworld, observables correlated in this way can't have definite or predetermined values before they are measured, so the property we attribute to one of the correlated quantum systems after a measurement couldn't have been there before the measurement. A quantum observable taking a particular value with a probability between 0 and 1 when measured is an intrinsically random event, not only for correlated observables in entangled quantum states, but in general for the values of observables in any quantum state (see section 4.3, *Really Random Qubits*, in Chapter 4, *Really Random*). This is a structural feature of quantum information, related to the way in which observables corresponding to properties of a system are related to other observables.

The weirdness of quantum correlations shows up in photon polarization measurements, where the outcomes can be 'horizontal' or 'vertical' for linear polarization in some direction, or electron spin measurements, where the outcomes can be 'up' or 'down' for spin in some direction. For definiteness, I'll stick to photon polarization rather than electron spin in the following, since that's likely to be more familiar to most readers. Choosing to measure the polarization of a photon in one of two directions  $45^\circ$  apart is like choosing to peel a banana by the stem end ( $S$ ) or the top end ( $T$ ), and the two possible tastes correspond to horizontal or vertical polarization in the respective directions.

The only really crucial thing you need to know about quantum mechanics to understand most of the discussion in this book is that quantum probabilities depend on the angle between the directions of polarization. Suppose you have a photon that is in a state of horizontal polarization in a certain direction, and you measure the polarization in a different direction at an angle  $\theta$  to the first direction. The probability of finding the photon horizontally polarized is  $\cos^2 \theta$ , and the probability of finding the photon vertically polarized is  $1 - \cos^2 \theta = \sin^2 \theta$ . The smaller the angle  $\theta$ , the closer the second direction is to the first direction, and the closer the probability of finding the photon to be horizontally polarized is to 1 ( $\cos^2 0$ ), which is what you'd expect.

For two separated photons in an entangled state called a Bell state that plays an important role in the following chapters, the two possible outcomes of measuring the polarization of one photon in some direction are equally probable. If you measure the polarizations of both photons in the same direction, you get the same outcome: either both photons are horizontally polarized in the direction of the polarization measurement, or they are both vertically polarized in a direction orthogonal to the direction of the polarization measurement, and the probability of each of these possibilities is  $1/2$ . Take this as a fact about photon polarization, a particular case of a general rule for quantum probabilities. As Feynman says: 'If you will simply admit that maybe [nature] does behave like this, you will find her a delightful entrancing thing.'

If you measure the polarizations of the two photons,  $A$  and  $B$ , in directions an angle  $\theta$  apart, the probability of getting the same outcome for both measurements is  $\cos^2 \theta$  (with equal probability that both photons are horizontally polarized in the directions of their respective measurements, or that they are both vertically polarized), and the probability of getting different outcomes is  $\sin^2 \theta$  (again with equal probability that  $A$  is horizontally polarized and  $B$  is vertically polarized, or that  $A$  is vertically polarized and  $B$  is horizontally polarized).

That's it. There's an introductory account of polarization and spin in Chapter 2, *Qubits*, and

more about how probabilities are calculated in the Supplement, *Some Mathematical Machinery*, at the end of the book, but the point of *Bananaworld* is to get at the heart of the conceptual revolution involved in the transition from classical to quantum physics by a more direct route than working through the formalism. What's really at issue is the nature of quantum correlations, and how these correlations can be exploited in information-theoretic tasks that are impossible with classical correlations. All this can be understood without getting bogged down in the mathematical machinery of quantum mechanics.



### The bottom line

- The connection between Bananaworld correlations and quantum correlations is through photon polarization measurements, where Alice and Bob can each choose to measure linear polarization in one of two directions, with possible outcomes 'horizontal' or 'vertical.' Quantum probabilities depend on the angle between the directions of polarization. If you have a photon that is in a state of horizontal polarization in a certain direction, and you measure the polarization in a direction at an angle  $\theta$  to the first direction, the probability of finding the photon horizontally polarized is  $\cos^2 \theta$ , and the probability of finding the photon vertically polarized is  $1 - \cos^2 \theta = \sin^2 \theta$ .
- For a perfectly correlated entangled state called a Bell state that plays an important role in the following chapters, if you measure the polarizations of both photons in the same direction, you get the same outcome: either both photons are horizontally polarized in the direction of the polarization measurement, or they are both vertically polarized in a direction orthogonal to the direction of the polarization measurement, with equal probability of  $1/2$  for these two alternatives.
- If you measure the polarizations of the photons in this Bell state in different directions an angle  $\theta$  apart, the probability of getting the same outcome for both measurements is  $\cos^2 \theta$  (with equal probability that both photons are horizontally polarized in the directions of their respective measurements, or that they are both vertically polarized), and the probability of getting different outcomes is  $\sin^2 \theta$  (again with equal probability for the two alternatives).

## 1.4 More

### 1.4.1 Special Relativity: The Basics

To amplify Feynman's remark that quantum mechanics is deeply puzzling in a way that the theory of relativity isn't, here's a brief account of special relativity, following Hermann Bondi's formulation in terms of his  $k$ -calculus.<sup>23</sup>

Don't be intimidated by the term. Bondi says:

I have discovered that what I call the  $k$ -calculus is still unknown in educated mathematical circles. In uneducated ones, of course, it is well-known. And I propose, therefore, to talk a little bit about it here, partly because I enjoy it, partly because I hope to put those of you who don't know it into a state where you can also easily derive the consequences of relativity. You will then see where it leads you and see that Einstein's principle of relativity gives perfectly reasonable results capable of being tested by experiment and observation.

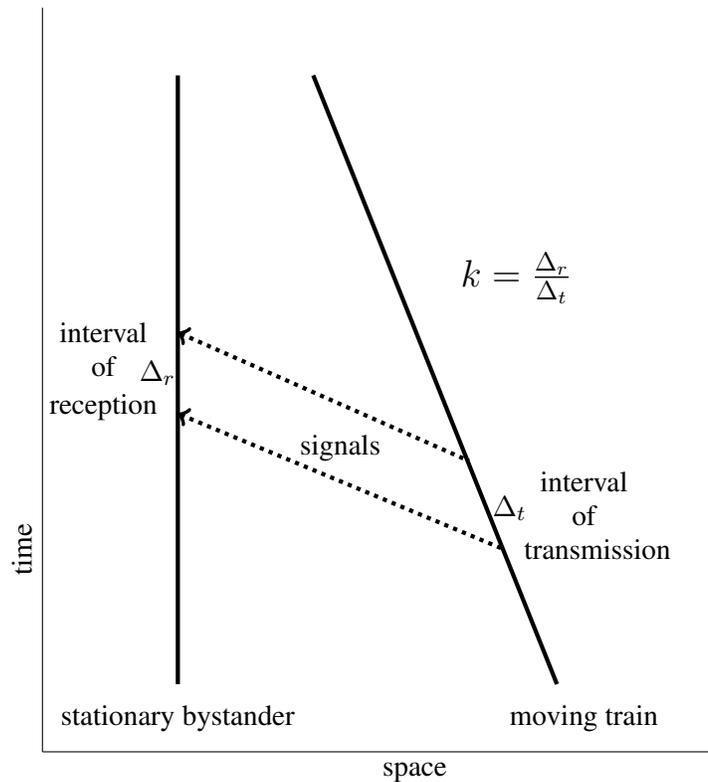


Figure 1.2: The  $k$ -factor:  $k = \Delta_r/\Delta_t$ . Adapted, with changes, from a diagram in H. Bondi, *Assumption and Myth in Physical Theory* (Cambridge University Press, Cambridge, 1967), p. 35.

The core insight of Bondi's formulation is the significance of the difference between the Doppler effect for sound and the Doppler effect for light. The Doppler effect for sound is the familiar increase in the pitch of a train's siren as it approaches a station, and the decrease in pitch as it recedes. Sound involves the vibration of air, the medium in which the sound waves propagate. The  $k$ -factor for a bystander at the station is the ratio of the interval of reception  $\Delta_r$  of the sound signals, corresponding to successive pressure peaks of the sound wave, to the interval of transmission  $\Delta_t$  by the moving train:  $k = \Delta_r/\Delta_t$ .

Figure 1.2 is a space-time diagram showing the relation between the two intervals. In these diagrams, time is in the vertical direction, and space in the horizontal direction. The thick vertical line illustrates a temporal sequence of events for the bystander at the same spatial point in the station. The thick sloping line illustrates a sequence of events in space and time for the train, which is moving towards the station, so the interval of reception is shorter than the interval of transmission. The dotted lines represent sound signals, moving from the train to the bystander. It should be clear from the diagram that the signals are moving faster than the train: as time goes by in the vertical direction, the signals cover more ground than the train in the horizontal spatial direction towards the bystander.

The same point can be made, without involving the physics of wave motion, by considering the Doppler effect when the signals are large objects, like cars moving along a highway, where the highway corresponds to the air as the stationary medium.

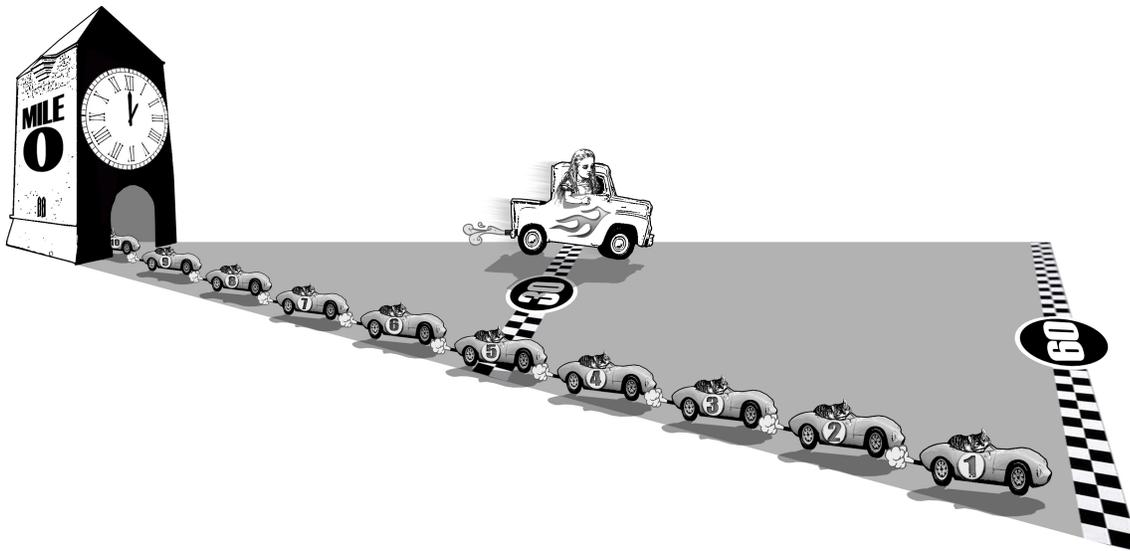


Figure 1.3: The Doppler effect for cars. The truck is receding from the parking garage, the stationary source of the car signals. The truck moves at 30 mph, the cars at 60 mph, beginning at noon. The illustration shows the situation at 1 pm. Here  $k_{rs} = \text{interval of reception/interval of transmission} = 12/6 = 2$ .

Consider a truck moving at 30 miles per hour leaving a stationary parking garage at 12 noon. Suppose 10 cars leave the parking garage at intervals of 6 minutes, traveling at 60 miles per hour towards the slower moving truck. The cars can be regarded as signals, with a signal velocity relative to the highway that is twice the velocity of the truck. The interval of transmission between car signals is 6 minutes. What is the interval of reception of these signals by the truck? If the last car leaves the parking garage at 1 pm when the truck has traveled 30 miles, it will reach the truck

## 1 Nobody Understands Quantum Mechanics

at 2 pm, when the truck has traveled an additional 30 miles and the car has traveled 60 miles in the hour between 1 pm and 2 pm. Since 10 cars pass the truck at equal time intervals over 120 minutes (the first car leaves at 12:06 pm and the last car leaves at 1 pm), the interval of reception of the car signals by the truck is 12 minutes. So  $k_{rs} = \frac{\Delta_r}{\Delta_t} = 12/6 = 2$ , where the  $r$  here indicates that the moving truck is *receding* from the stationary parking garage, and the  $s$  indicates that the signal source is *stationary*. See Figure 1.3.

Now suppose the truck approaches the parking garage, beginning 30 miles away at noon, and the car signals leave the parking garage and travel towards the moving truck as it approaches the parking garage. If the first car leaves the parking garage at noon traveling at 60 miles per hour, it will reach the truck at the 20 mile mark, when the slower-moving truck has traveled 10 miles, so at 12:20 pm. Since 10 cars pass the truck in the 40 minutes between 12:20 pm and 1:00 pm when the truck arrives at the parking garage, the interval of reception is 4 minutes, and  $k_{as} = 4/6 = 2/3$ , where the  $a$  here indicates that the moving truck is *approaching* the stationary parking garage. See Figure 1.4.

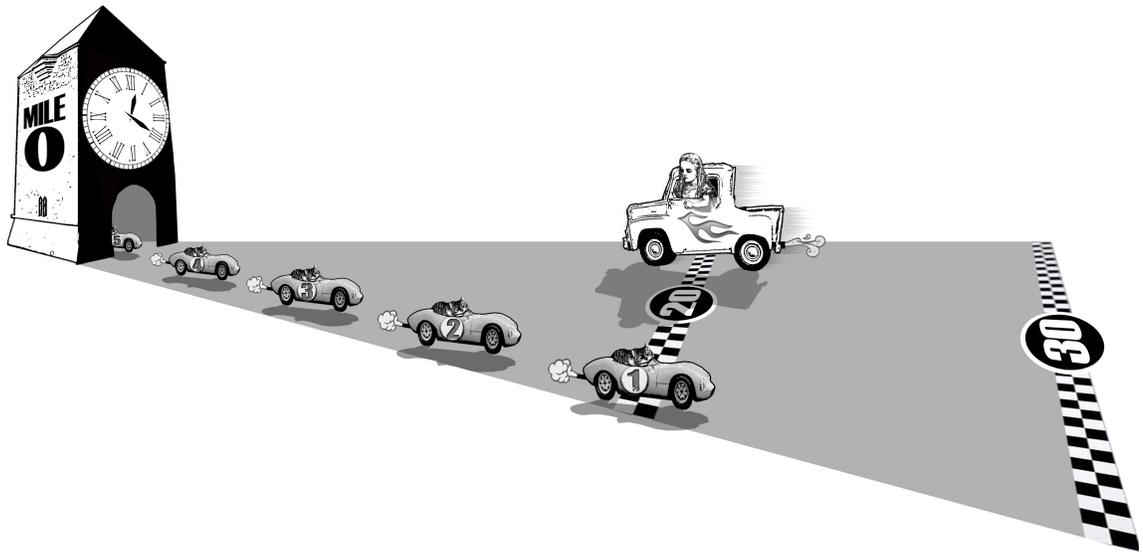


Figure 1.4: The Doppler effect for cars. The truck is approaching the parking garage, the stationary source of the car signals. The truck moves at 30 mph, beginning at the 30 mile mark at noon. The cars move at 60 mph. The illustration shows the situation at 12:20 pm. Here  $k_{as} = \text{interval of reception/interval of transmission} = 4/6 = 2/3$ .

In both these case,  $k_{rs}$  and  $k_{as}$ , the source of the car signals is the *stationary* parking garage. There are two other cases to consider,  $k_{rm}$  and  $k_{am}$ , where the source is the *moving* truck, either receding from or approaching the parking garage.

For the *receding* case, suppose the truck leaves the parking garage at noon traveling at 30 miles

per hour. Suppose 10 cars are parked along the highway at 3 mile intervals from the parking garage. Every 6 minutes the moving truck passes a car, and as it passes, the car begins traveling towards the parking garage at 60 miles per hour. The cars can be regarded as signals from the moving truck (like the sound signals from a moving train's siren, which produces a wave vibration in the stationary air as the train passes). If the last car begins traveling towards the parking garage at 1 pm, after the truck has traveled 30 miles, it will take half an hour to cover the 30 miles to the parking garage, so it will arrive at the parking garage at 1:30 pm. Since 10 cars arrive at equal time intervals over 90 minutes, the interval of reception of the car signals at the parking garage is 9 minutes, and  $k_{rm} = 9/6 = 3/2$ . See Figure 1.5.

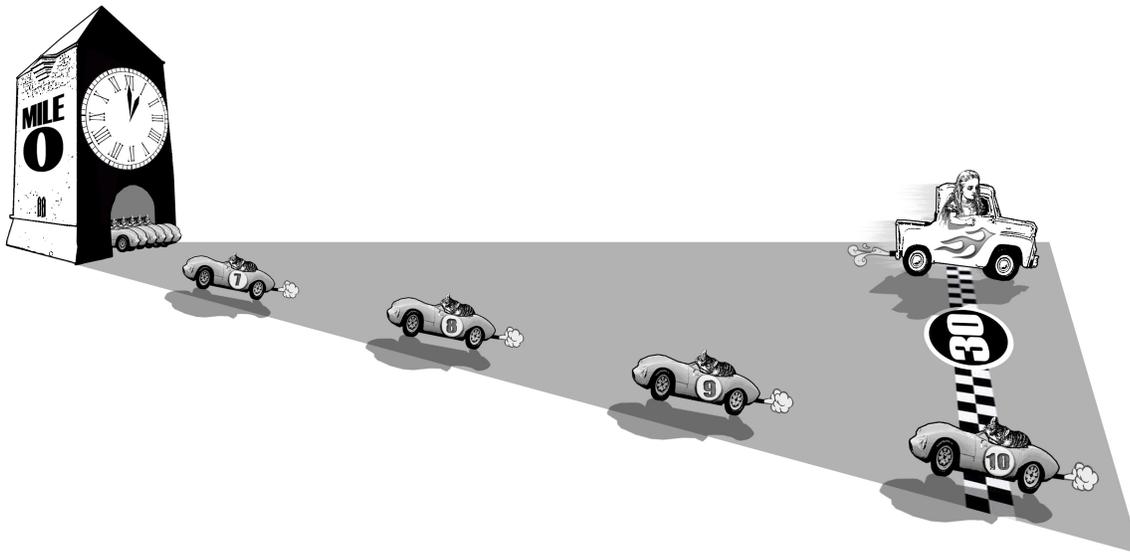


Figure 1.5: The Doppler effect for cars. The truck is receding from the parking garage, and the car signals begin to move towards the parking garage as the truck passes, so the source of the signals is the moving truck. The truck moves at 30 mph, the cars at 60 mph, beginning at noon. The illustration depicts the situation at 1:00 pm. Here  $k_{rm} = \text{interval of reception/interval of transmission} = 9/6 = 3/2$ .

For the *approaching* case, suppose the truck approaches the parking garage, beginning 30 miles away at noon. If 10 cars are parked along the highway at 3 mile intervals from the parking garage, and the first car begins traveling towards the parking garage at noon at 60 miles per hour, it will reach the parking garage after half an hour, at 12:30 pm. Since 10 cars arrive in the 30 minutes between 12:30 pm and 1:00 pm when the truck arrives at the parking garage, the interval of reception is 3 minutes, and  $k_{am} = 3/6 = 1/2$ . See Figure 1.6.

So there are four  $k$ -factors for signals passing between a stationary system and a moving system: two  $k$ -factors when the moving system is receding from the stationary system, depending on

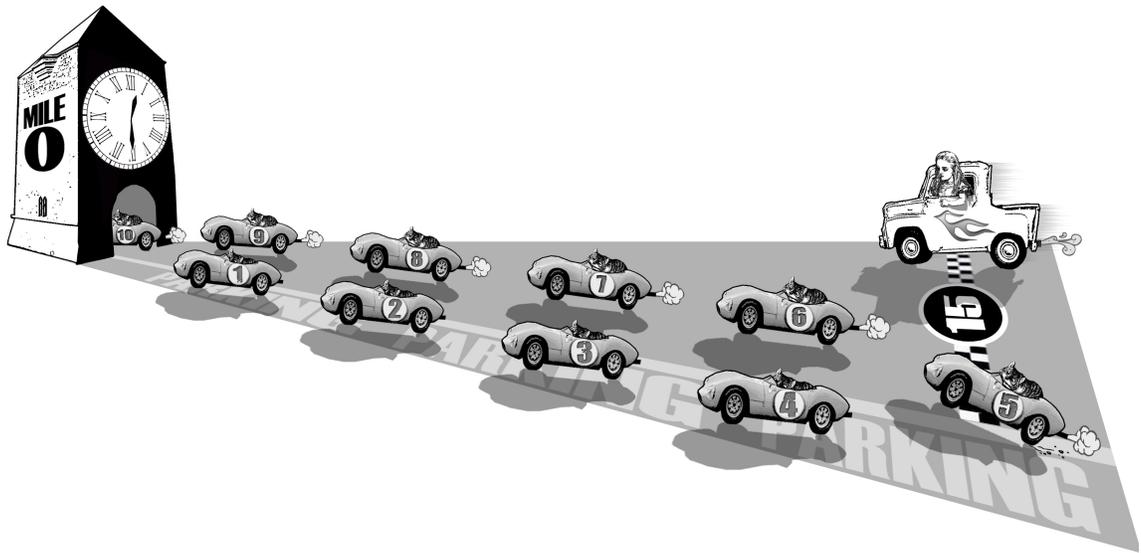


Figure 1.6: The Doppler effect for cars. The truck is approaching the parking garage, and the car signals begin to move towards the parking garage as the truck passes, so the source of the signals is the moving truck. The truck moves at 30 mph, beginning at the 30 mile mark at noon. The cars move at 60 mph. The illustration depicts the situation at 12:30 pm. Here  $k_{am} = \text{interval of reception}/\text{interval of transmission} = 3/6 = 1/2$ .

whether the signal source is moving ( $k_{rm} = 3/2$ ) or stationary ( $k_{rs} = 2$ ), and two  $k$ -factors when the moving system approaches the stationary system, depending on whether the signal source is moving ( $k_{am} = 1/2$ ) or stationary ( $k_{as} = 2/3$ ). (The values of  $k$  also depend on the ratio of the speed of the moving system to the signal speed, so these values would change for different truck speeds and car speeds.) Notice that  $k_{am} = 1/k_{rs}$  and  $k_{as} = 1/k_{rm}$ , which is true in general for a moving system approaching or receding from a stationary system at the same speed.

Now here's the punchline: *for light, there is no stationary medium for the transmission of light signals corresponding to the stationary highway with respect to which rest and motion are defined for the stationary parking garage, the car signals, and the moving truck.* For sound signals, the air corresponds to the highway as the stationary medium in which sound waves propagate at around 760 miles per hour at sea level. The theory of special relativity drops the idea of a 'luminiferous ether' as the medium of propagation for light signals—an idea proposed by Lorentz in his rival theory.

Aristotle's physics distinguished a state of rest from other states of motion, but Galileo and Newton realized that a particular set of states of motion, the 'inertial' states of motion, are all equivalent and distinguished from other states of motion. Inertial systems are systems moving with constant velocity relative to each other, as distinct from accelerating systems. Simply put: velocity

is relative, but acceleration is absolute.

The theory of special relativity rests on two principles: the light postulate, that there is ‘no overtaking of light by light,’<sup>24</sup> as Bondi puts it, and the relativity principle, which Bondi sums up as the slogan ‘velocity doesn’t matter.’<sup>25</sup> The light postulate says that the velocity of light in empty space is constant, independent of the motion of the source and the same for all inertial observers. So ‘no overtaking of light by light’ means, in particular, no overtaking of light by anything else: the relative velocity of light is the same for differently moving inertial observers, quite unlike the relative velocity of sound or cars. The relativity principle is an extension of the Galilean or Newtonian principle of relativity from mechanical phenomena to all phenomena, in particular to electromagnetic phenomena such as light. The statement is simply that physics is the same for all inertial systems, so nothing in the way physical systems behave could tell you how you are moving if you are in an enclosed space like a spaceship, provided the spaceship is not accelerating or rotating.

It follows that for light signals passing between two systems moving away from each other with constant relative velocity,  $k_{rm} = k_{rs} = k_r$ , because there is no fact of the matter about which system is ‘really’ at rest and which system is moving: all we have is that the two systems are receding from each other at a certain constant relative velocity. Similarly,  $k_{am} = k_{as} = k_a$ .

### The bottom line

- In Bondi’s  $k$ -calculus, the  $k$ -factor is the ratio of the interval of reception of signals to the interval of transmission.
- In a classical (Newtonian) theory of space and time, where it makes sense to think of motion relative to a state of rest (relative to the earth for sound signals in the air, or relative to a hypothetical ‘luminiferous ether’ for light signals), there are four  $k$ -factors for signals passing between a stationary system and a moving system: two  $k$ -factors when the moving system is receding from the stationary system, depending on whether the signal source is moving ( $k_{rm}$ ) or stationary ( $k_{rs}$ ), and two  $k$ -factors when the moving system approaches the stationary system, depending on whether the signal source is moving ( $k_{am}$ ) or stationary ( $k_{as}$ ). For the car and truck example in the text,  $k_{rm} = 3/2$ ,  $k_{rs} = 2$ ,  $k_{am} = 1/2$ , and  $k_{as} = 2/3$ .
- Special relativity rests on two principles: the light postulate, ‘no overtaking of light by light,’ and the relativity principle, ‘velocity doesn’t matter’ for both mechanical phenomena and electromagnetic phenomena involving light. There’s no state of motion relative to a state of absolute rest, only relative velocities between systems.
- It follows that for light signals passing between two systems moving away from each other with constant relative velocity,  $k_{rm} = k_{rs} = k_r$  and  $k_{am} = k_{as} = k_a$ , because there is no fact of the matter about which system is ‘really’ at rest and which system is moving. So there are only two  $k$ -factors,  $k_r$  and  $k_a$ , and  $k_r = 1/k_a$ . That makes all the difference.

### 1.4.2 The Lorentz Transformation

You might suspect that the  $k$ -factor for two systems approaching each other at a certain speed,  $k_a$ , is the reciprocal of the  $k$ -factor for two systems receding from each other with the same speed,  $k_r$ , and this is so. Here's the argument.

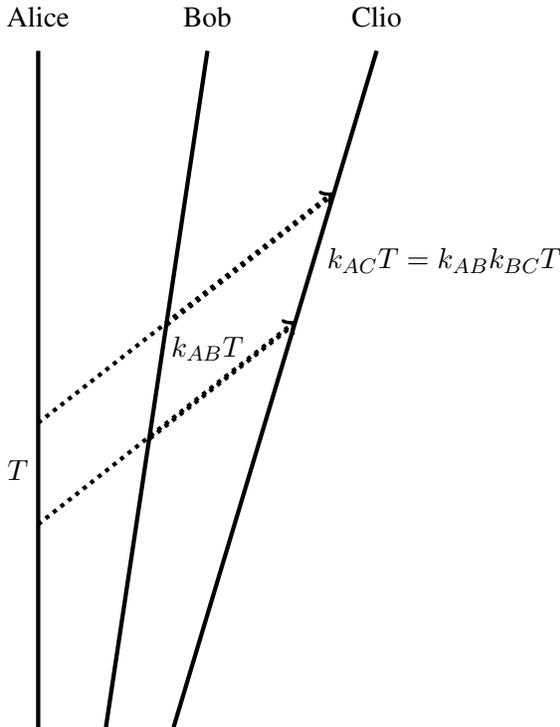


Figure 1.7: Composition of  $k$ -factors. As in Figure 1.2, only one direction of space is shown. The time axis is in the vertical direction and the space axis is in the horizontal direction. Adapted, with changes, from a diagram in H. Bondi, *Assumption and Myth in Physical Theory* (Cambridge University Press, Cambridge, 1967), p. 40.

Two inertial systems are associated with a specific  $k$ -value that is a function of their relative velocity. In particular, by the relativity principle, the same  $k$ -factor applies if Alice is the source of signals to Bob, or if Bob is the source of signals to Alice, and the velocity of light is the same from Alice to Bob as from Bob to Alice. Now suppose that Alice, Bob, and Clio are three inertial observers equipped with identical clocks moving in a straight line with different relative velocities so that the  $k$ -factor is  $k_{AB}$  between Alice and Bob, and  $k_{BC}$  between Bob and Clio. See Figure 1.7. If Alice sends light signals to Bob at regular intervals  $T$  by her clock, the signals will be received by Bob at intervals  $k_{AB}T$  by his clock. Suppose that when Bob receives Alice's signals, he immediately sends light signals to Clio. Bob's signals will be received by Clio at intervals  $k_{BC}(k_{AB}T)$  by Clio's clock. If Alice's signals continue traveling towards Clio, they will be received by Clio

simultaneously with Bob's signals, since there is no overtaking of light by light. It follows that if the  $k$ -factor between Alice and Clio is  $k_{AC}$ , then  $k_{AC}T = k_{BC}k_{AB}T$ , and so  $k_{AC} = k_{AB}k_{BC}$ . If Alice and Clio are stationary with respect to each other, and Bob moves from Alice towards Clio, then  $k_{AC} = 1$  (the interval of reception is the same as the interval of transmission), and so  $k_{BC} = 1/k_{AB}$ .

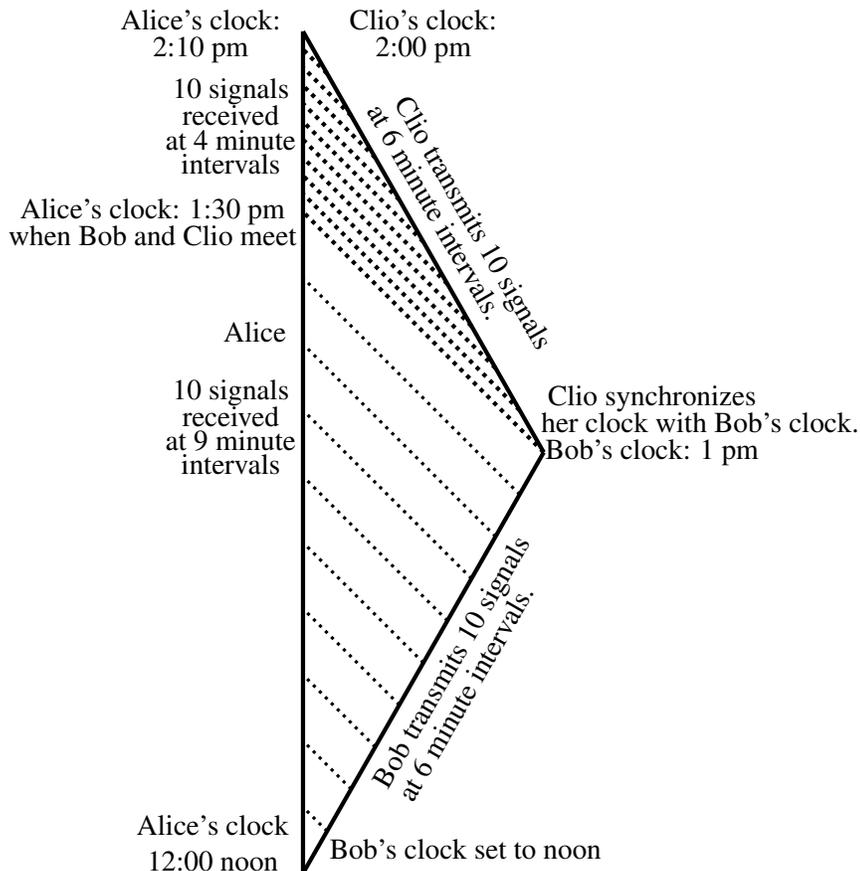


Figure 1.8: Time dilation for Alice relative to Bob and Clio. As in the previous space-time diagrams, only one direction of space is shown. The time axis is in the vertical direction and the space axis is in the horizontal direction. Adapted, with changes, from a diagram in H. Bondi, *Relativity and Common Sense* (Heinemann Educational Books, London, 1965), p. 84.

Suppose that Alice and Bob are inertial observers moving relative to each other and that  $k_{AB} = 3/2$ . Suppose Bob passes Alice at noon, receding from Alice in a certain direction, and that they synchronize their clocks as they pass at noon. (See Figure 1.8.) Bob sends Alice ten light signals six minutes apart. Alice receives the signals at intervals  $k_{AB} \cdot 6 = 9$  minutes, so the last signal

reaches her at 1:30 pm by her clock. At 1 pm by Bob’s clock, Clio passes Bob moving towards Alice at the same speed that Bob recedes from Alice. Bob and Clio synchronize their clocks as they pass at 1 pm. Clio now sends Alice ten light signals six minutes apart. Since  $k_{CA} = 1/k_{AB} = 2/3$ , Alice receives the signals at intervals of four minutes, so the last signal arrives 40 minutes after 1:30 pm at 2:10 pm by her clock—just as Clio passes and registers 2:00 pm by her clock! Astonishingly, the time measured by Alice’s clock between two events (meeting Bob and meeting Clio) is a longer time interval than the combined times measured by Bob’s clock and Clio’s clock, even though all three clocks are identical and synchronized. This phenomenon is known as time dilation: clocks moving relative to Alice run slow relative to Alice’s clock.

Where did the extra ten minutes come from? As puzzling as this may seem at first sight, the time difference is an immediate consequence of the assumptions about relativistic  $k$ -factors, which follow from the light postulate (no overtaking of light by light) and the relativity principle (the state of motion doesn’t matter for different inertial systems: the physics is the same). You don’t get the difference in time measurements with cars as signals. In that case there are four  $k$ -factors and, for receding Bob and approaching Clio who are the source of signals to Alice, the relevant  $k$ -factors are  $k_{rm} = 3/2$  for Alice and Bob, and  $k_{am} = 1/2$  for Clio and Alice. Then Alice receives Bob’s signals at intervals of nine minutes as before, so that the last signal arrives at 1:30 pm, but she receives Clio’s signals at intervals of three minutes, so the last signal arrives at 2 pm, just as Clio passes.

The essential point here is that there is no universal highway for signals in our universe. We can suppose that Alice, Bob, and Clio are equipped with identical clocks, and we can suppose that clocks can be synchronized when two physical systems are momentarily in the same place. But we can’t assume that clocks run at the same rate when they are in different parts of space moving relative to each other. The only way to relate the times of moving clocks in separate parts of space is for Alice, Bob, and Clio to communicate by signaling to each other, and the only general way to do this is via light signals, or electromagnetic radiation, for which there is no medium or universal highway that could define an absolute state of rest.

The relation between the space and time coordinates of different inertial observers in special relativity is known as the Lorentz transformation. It’s straightforward to derive the Lorentz transformation in Bondi’s  $k$ -calculus. Time dilation, length contraction, and the relativity of simultaneity—features of Minkowski space-time—all follow from the Lorentz transformation.

First, what’s the relation between the  $k$ -factor and relative velocity? Suppose, for definiteness, that Alice and Bob are receding from each other along a straight line. For convenience, you can think of Alice at rest and Bob as moving, but you could equally well think of Bob at rest and Alice as moving away from Bob. Suppose Alice and Bob synchronize their clocks at zero as they pass each other. At a time  $T$  later, Alice sends Bob a light signal, which is received by Bob at the time  $kT$  by his clock. Bob immediately reflects Alice’s light signal back to her. Alice receives the returning signal at the time  $k(kT)$  by her clock and calculates the round-trip time taken by the signal to cover the distance from her to Bob and from Bob back to her as  $k^2T - kT = (k^2 - 1)T$  by her clock. See Figure 1.9.

To figure out the distance between her and Bob at the moment her light signal is reflected by Bob, Alice multiplies the time taken for the light to travel this distance by the speed of light. She

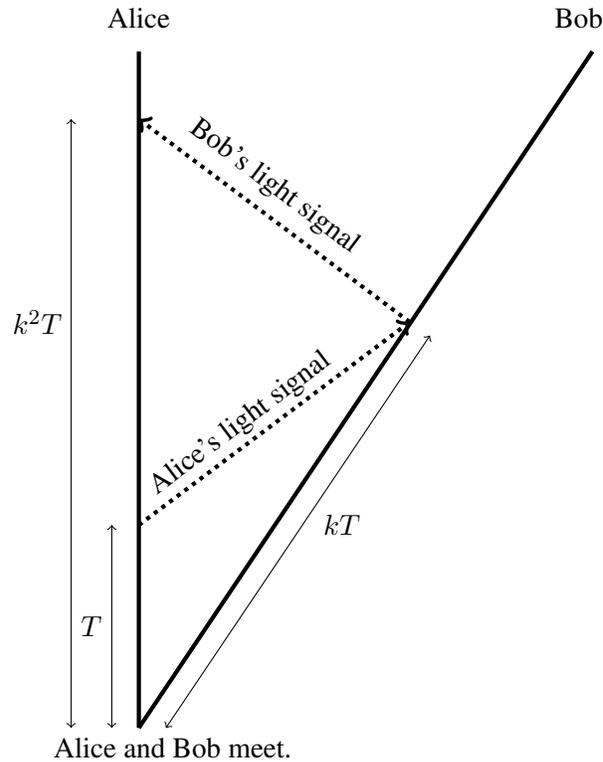


Figure 1.9: The relation between the  $k$ -factor and relative velocity. As in the previous space-time diagrams, only one direction of space is shown. The time axis is in the vertical direction and the space axis is in the horizontal direction. Adapted, with changes, from a diagram in H. Bondi, *Assumption and Myth in Physical Theory* (Cambridge University Press, Cambridge, 1967), p. 38.

calculates the time as half the round-trip time,  $1/2(k^2 - 1)T$ . The speed of light is about 300,000 kilometers per second. It's convenient to take the unit of distance as the distance light covers in a second, the unit of time. Then the speed of light is 1 (one unit of distance per one unit of time). In these units, the distance is  $1/2(k^2 - 1)T$ . Alice has no way of measuring the time of this remote event, and no direct access to the time registered by Bob's clock for this event, so she assigns the time  $T + 1/2(k^2 - 1)T = 1/2(k^2 + 1)T$  to the event, half-way between the time she transmitted her signal and the time she received Bob's signal. This is the time taken by Bob to cover the distance  $1/2(k^2 - 1)T$ , so Bob's velocity is:

$$v = \frac{k^2 - 1}{k^2 + 1}$$

or

$$k = \sqrt{\frac{1 + v}{1 - v}}$$

## 1 Nobody Understands Quantum Mechanics

which is always less than 1, the speed of light.

If Alice is walking in a moving train, the velocity of Alice relative to the earth is the velocity of Alice relative to the train plus the velocity of the train relative to the earth, according to the classical or Galilean law of composition of velocities

$$v_{AC} = v_{AB} + v_{BC}$$

where  $A$  here refers to Alice,  $B$  to the train, and  $C$  to the earth. The corresponding relativistic law follows from  $k_{AC} = k_{AB}k_{BC}$  by substituting the appropriate expressions for the relative velocities in the  $k$ -factors:

$$v_{AC} = \frac{v_{AB} + v_{BC}}{1 + v_{AB}v_{BC}}$$

Even if  $v_{AB}$  and  $v_{BC}$  are both very close to 1, the velocity of light,  $v_{AC} < 1$ , and if  $v_{AB} = v_{BC} = 1$ ,  $v_{AC} = 1$  (no overtaking of light by light).

To derive the Lorentz transformation, first note that if Alice transmits a light signal at time  $t - x$  by her clock and the light signal is reflected by some event  $E$  and reaches her at time  $t + x$  by her clock, she will assign space and time coordinates  $x, t$  to the event  $E$ , as she did in the derivation of the relation between  $k$  and relative velocity. See Figure 1.10. (That's because the time taken by Alice's light signal to travel from Alice to  $E$  and back to Alice is  $(t + x) - (t - x) = 2x$ . So Alice assigns a distance  $x$  to the event  $E$ , taking the velocity of light as 1 and the time it takes the signal to travel to  $E$  as half the round-trip time,  $x$ . She assigns a time  $t = (t - x) + x$  to  $E$ , which is half-way between the transmission time and the reception time.) Suppose Bob is moving away from Alice and they synchronize clocks at zero as they pass each other. If Bob transmits a light signal towards the event  $E$  just as Alice's signal passes him at time  $t' - x'$  by his clock, and the signal is reflected back to Bob by  $E$  together with Alice's signal and reaches him at time  $t' + x'$  by his clock, Bob will assign coordinates  $x', t'$  to  $E$ .

Now,  $t' - x' = k(t - x)$  and  $t + x = k(t' + x')$ . Substitute  $\sqrt{\frac{1+v}{1-v}}$  for  $k$  and the two equations simplify to the Lorentz transformation:

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - v^2}} \\ t' &= \frac{t - vx}{\sqrt{1 - v^2}} \end{aligned}$$

Length contraction and time dilation follow from the Lorentz transformation. If Bob is moving relative to Alice and assigns the primed space coordinates 0 and  $L$  to the beginning and end of a ruler, then Alice will assign the beginning and end of the ruler the unprimed space coordinates  $vt$  and  $vt + L\sqrt{1 - v^2}$ , and so a shorter length  $L\sqrt{1 - v^2}$  at time  $t$  by her clock. Similarly, if Bob assigns the primed time coordinates 0 and  $T$  to two events, Alice will assign the unprimed time coordinates  $t = vx$  and  $t = T\sqrt{1 - v^2} + vx$  to the events, and so a shorter time interval  $T\sqrt{1 - v^2}$  at the same place  $x$  by her reckoning. So Bob's moving clock will appear to Alice to run slow.

The relativity of simultaneity also follows from the Lorentz transformation. Suppose Alice assigns a time  $t$  by her clock to two events at different places,  $x_1$  and  $x_2$ . Bob will assign different

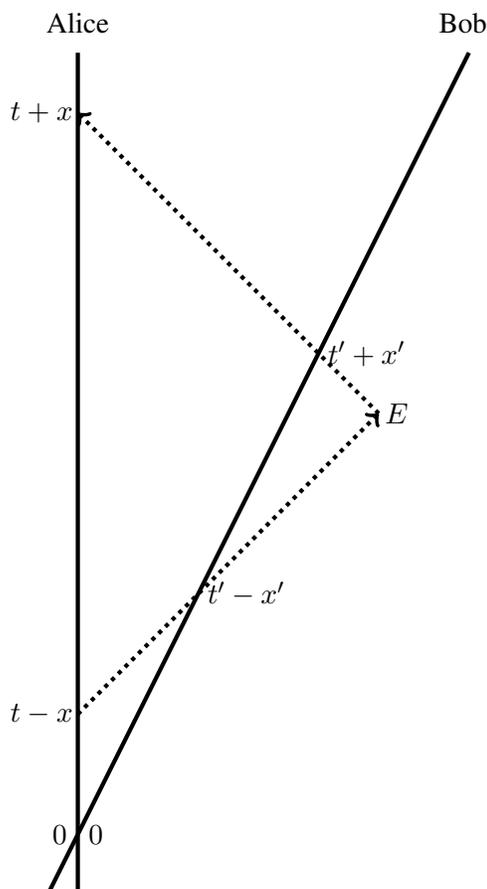


Figure 1.10: The Lorentz transformation. As in the previous space-time diagrams, only one direction of space is shown. The time axis is in the vertical direction and the space axis is in the horizontal direction. Adapted, with changes, from a diagram in H. Bondi, *Relativity and Common Sense* (Heinemann Educational Books, London, 1965), p. 117.

times  $t'_1 = \frac{t-vx_1}{\sqrt{1-v^2}}$  and  $t'_2 = \frac{t-vx_2}{\sqrt{1-v^2}}$  to the events by his clock. So there is a time difference of  $t'_2 - t'_1 = \frac{v(x_1-x_2)}{\sqrt{1-v^2}}$  for Bob between two events that are simultaneous for Alice. If  $x_2 > x_1$  and  $v$  is positive,  $t'_2 - t'_1$  is a negative number, so  $t'_1 > t'_2$ , which means that Bob sees event  $E_1$  occurring *after*  $E_2$ , while Alice sees the two events occurring at the same time.

Events in relativistic (Minkowski) space-time can be divided into three sets. See Figure 1.11. Events that can be connected to an event  $O$  by a light ray are said to be lightlike separated from  $O$ . These are the events that lie on the 'light cone' in 4-dimensional space-time (here only one dimension of space is shown), represented by the diagonal lines at a  $45^\circ$  angle (because the velocity of light is conventionally taken as 1, which means that light covers one unit of distance in the horizontal direction in one unit of time in the vertical direction). Events that can be connected to

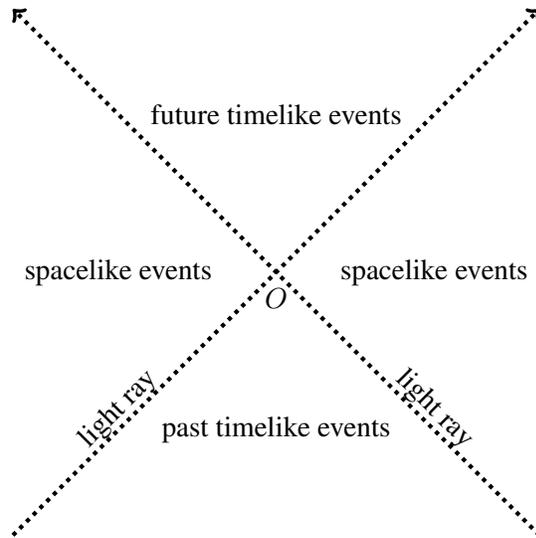


Figure 1.11: The light cone defined by light rays divides events into three sets. Events represented by points on the light cone are lightlike separated from  $O$ : they can be connected to  $O$  by a light ray. Events represented by points in the top and bottom regions of the light cone are timelike separated from  $O$ : they can be connected to  $O$  by a signal traveling slower than light. Events represented by the points in the left and right regions of the diagram, outside the light cone, are spacelike separated from  $O$ : they can't be connected to  $O$  by a signal traveling at or less than the speed of light. As in the previous space-time diagrams, only one direction of space is shown. The time axis is in the vertical direction and the space axis is in the horizontal direction.

$O$  by a signal traveling slower than light are said to be timelike separated from  $O$ . These are the events represented by the points in the top and bottom regions of the light cone. A line drawn from  $O$  to any event in these regions will have a slope of more than  $45^\circ$ , indicating a velocity of less than 1 (in one unit of time in the vertical direction, less than one unit of distance is covered in the horizontal direction). Events that can't be connected to  $O$  by a signal traveling at or less than the speed of light are said to be spacelike separated from  $O$ . These are the events represented by the points in the left and right regions of the diagram, outside the light cone. A line drawn from  $O$  to any event in these regions will have a slope less than  $45^\circ$ , indicating a velocity of more than 1 (in one unit of time in the vertical direction, more than one unit of distance is covered in the horizontal direction).

If two events are spacelike separated, the time order can be different for different inertial observers. From the Lorentz transformation

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - v(x_2 - x_1)}{\sqrt{1 - v^2}}$$

For spacelike separated events  $E_1$  and  $E_2$ , the time difference between the two events is smaller

than the space difference (in units in which the velocity of light is 1). The relative velocity of two observers,  $v$ , is less than or equal to 1, the velocity of light. For sufficiently small values of  $v$ ,  $t_2 - t_1$  will be greater than  $v(x_2 - x_1)$ , but for values of  $v$  sufficiently close to 1,  $t_2 - t_1$  will be smaller than  $v(x_2 - x_1)$ . So you could have  $t_2 - t_1$  positive but  $t_2 - t_1 - v(x_2 - x_1)$  negative, which means that  $t'_2 - t'_1$  could be negative while  $t_2 - t_1$  is positive. In other words,  $E_2$  could occur later than  $E_1$  for Alice, but earlier than  $E_1$  for Bob.

For timelike separated events, the time difference between two events is always greater than the space difference, so you can't have  $t_2 - t_1 < v(x_2 - x_1)$  for any value of  $v$  less than or equal to 1, and the time order is the same for all inertial observers.

The aim of this subsection was to show that, even if the nature of space and time remains mysterious in some deep metaphysical sense, the puzzle about how space and time could be different for Alice and Bob is resolved once we see that time dilation and length contraction follow from the two physically motivated principles of Einstein's theory: the light postulate and the principle of relativity. 'Nobody understands quantum mechanics' because there's no agreement among physicists on a similar analysis for quantum phenomena. We don't see that quantum phenomena must be the way they are because of some basic physical principles.

Some of the concepts introduced in this subsection are relevant to later sections, in particular the notion of spacelike separated events, for which the time order can be different for different inertial observers.



### The bottom line

- Inertial systems are systems moving with constant velocity relative to each other, as distinct from accelerating systems. The relation between the space and time coordinates of different inertial observers in special relativity follows from the  $k$ -calculus and is known as the Lorentz transformation.
- Length contraction, time dilation, and the relativity of simultaneity—features of relativistic (Minkowski) space-time—all follow from the Lorentz transformation.
- Events in relativistic space-time can be divided into three sets: lightlike (events that can be connected to an event  $O$  by a light ray), timelike (events that can be connected to  $O$  by a signal traveling slower than light, so events inside the forward or backward light cones of  $O$ ), and spacelike (events that can't be connected by a signal traveling at or less than the speed of light, so events outside the light cone of  $O$ ).
- For timelike separated events, the time order is the same for all inertial observers, but for spacelike separated events, the time order can be different for different inertial observers.

## Notes

1. Richard P. Feynman, Julian Schwinger, and Sin-Itiro Tomonago were joint winners of the 1965 Nobel Prize ‘for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles.’ The quotation ‘nobody understands quantum mechanics’ is from Richard Feynman: *The Character of Physical Law* (MIT Press, Cambridge, 1967), p. 129. The book is a transcript of the Messenger Lectures that Feynman gave at Cornell University in 1964.
2. The title of Einstein’s historic paper on the theory of special relativity is ‘On the electrodynamics of moving bodies’ (‘Zur Elektrodynamik bewegter Körper,’ *Annalen der Physik*, 17, 891–921 (1905)). Einstein was awarded the Nobel Prize in 1922 for his discovery of the photoelectric effect and ‘for his services to Theoretical Physics.’ Since there was no award for physics in 1921, Einstein’s award in 1922 was officially the Nobel prize in physics for 1921. Curiously, the Nobel committee did not regard the theory of relativity as worth a Nobel prize.
3. Minkowski’s comment that ‘space by itself, and time by itself, are doomed to fade away into mere shadows’ is from Minkowski’s article ‘Space and time,’ in W. Perrett and G. Jefferey (eds.), *The Principle of Relativity* (Dover, New York, 1952), p. 75.
4. Niels Bohr’s theory of the atom was published in a two-part paper ‘On the constitution of atoms and molecules,’ in *Philosophical Magazine* 26, 1–24, 476–502 (1913). Arnold Sommerfeld published an improvement and extension of the theory in a two-part paper ‘Zur Quantentheorie der Spektrallinien’ in *Annalen de Physik* 51, 1–94, 125–167. The full theory is referred to as the Bohr-Sommerfeld theory.
5. Werner Heisenberg’s 1925 break-through paper, ‘Über Quantentheoretischer Umdeutung kinematischer und mechanischer Beziehungen,’ was published in *Zeitschrift für Physik* 33, 879–893 (1925). In the same year, Max Born and Pascual Jordan published the first part of a two-part paper ‘Zur Quantenmechanik’ in *Zeitschrift für Physik* 34, 858–888 (1925). Part II of this paper, referred to by historians as the ‘three-man paper,’ was co-authored with Heisenberg and published in *Zeitschrift für Physik* 35, 557-615 (1926).
6. The expression ‘saving the appearances’ has its origin in Osiander’s preface to Copernicus’ 1543 heliocentric theory *De Revolutionibus*, proposed as a rival to Ptolemy’s geocentric theory. To avoid censure by the Catholic Church, Osiander presented Copernicus’ theory as a mathematical model that ‘saves the appearances,’ in the sense that it was empirically adequate in representing astronomical phenomena—in fact, superior in some ways to Ptolemy’s theory in this respect—without purporting to present a true picture of the relationship between the earth, the sun, and the planets. Osiander’s position is now known as ‘instrumentalism,’ the view that a scientific theory is an instrument for prediction, with the sole aim of being empirically adequate to the phenomena, not necessarily a true account of the way things are.
7. Heisenberg’s comment about all his efforts going toward killing off and replacing the electron orbits is in a letter Heisenberg wrote to Wolfgang Pauli dated July 9, 1925. The remark is

quoted in David C. Cassidy, *Uncertainty: The Life and Science of Werner Heisenberg* (W.H. Freeman, New York, 1992), p. 197.

8. Erwin Schrödinger's papers on wave mechanics were published as 'Quantisierung als Eigenwertproblem,' *Annalen der Physik* 79, 361–376 (1926) and 'An undulatory theory of the mechanics of atoms and molecules,' *Physical Review* 28, 1049–1070 (1926). His proof of the equivalence of his wave mechanics with Heisenberg's matrix mechanics was published as 'Über das Verhältnis der Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen,' *Annalen der Physik* 79, 734–756 (1926).
9. Schrödinger gave his Royal Institution (London) lecture in 1928. It was published as 'Four lectures on wave mechanics,' in *Collected Papers on Wave Mechanics* (Chelsea Publishing Company, New York, 1982). The quotation about a wave as a representation of quantum reality being 'merely an adequate mathematical description of what happens' is on p. 160.
10. Paul Dirac's formalization of quantum mechanics is developed in his book *The Principles of Quantum Mechanics* (Clarendon Press, Oxford, 1958).
11. Von Neuman's more rigorous 1932 formalization is set out in his book *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, 1955).
12. Von Neuman's comment on probabilities in quantum mechanics being 'perfectly new and *sui generis* aspects of physical reality' is from 'Quantum logics: strict- and probability-logics,' a 1937 unfinished manuscript published in A.H. Taub (ed.), *Collected Works of John von Neumann IV* (Pergamon Press, Oxford and New York, 1961), pp. 195–197.
13. Erwin Schrödinger's comment about entanglement being '*the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought' is from his article 'Discussion of Probability Relations Between Separated Systems,' *Proceedings of the Cambridge Philosophical Society*, 31, 555–563 (1935). the quotation is on p. 555.
14. Claude Shannon's seminal two-part paper on information theory: 'A mathematical theory of communication,' *The Bell System Technical Journal* 27, 379–423, 623–656 (1948). The statement about the semantic aspects of communication being irrelevant to the engineering problem is on p. 379.
15. John Wheeler's slogan 'it from bit' is from J. A. Wheeler, 'Information, physics, quantum: the search for links,' in W. Zurek (ed.), *complexity, Entropy and the Physics of information* (Addison-Wesley, Redwood City, CA, 1990), pp. 3–28.
16. Rolf Landauer's comment that 'information is physical' is from his article 'information is physical,' *Physics Today* 44, 23–29 (1991). See also R. Landauer, 'The physical nature of information,' *Physics Letters A* 217, 188–193 (1996), and 'Information is a physical entity,' *Physica A* 263, 63–67 (1999). From Landauer's writings, there's no reason to think that 'information is physical' should be understood as the claim that information is primary and that

‘stuff’ is in some sense derived from information, although Landauer’s slogan has been cited to support such claims.

17. The quotation from Vlatko Vedral’s book *Decoding Reality* (Oxford University Press, Oxford, 2010) that ‘our reality is ultimately made up of information’ is on p. 13. On p. 215, he writes: ‘This book has argued that everything in our reality is made up of information.’ The statement that ‘the laws of Nature are information about information’ is on p. 218.
18. For a rival view, diametrically opposed to Vedral’s, but also at odds with the information-theoretic interpretation of quantum mechanics I put forward in this book, see Christopher Timpson’s book, *Quantum Information Theory and the Foundations of Quantum Mechanics* (Oxford University Press, Oxford, 2013).
19. John Bell’s comment ‘*Information? Whose information? information about what?*’ is from his article ‘Against measurement,’ in *Physics World* 8, 33–40 (1990). The comment is on p. 34. It is reprinted in A. Miller (ed.) *Sixty-Two Years of Uncertainty: Historical, Philosophical and Physical Inquiries into the Foundations of Quantum Mechanics* (Plenum, New York, 1990), pp. 17–31.
20. Sandu Popescu and David Rohrlich introduced the nonlocal box now referred to as a PR box in their paper ‘Quantum nonlocality as an axiom,’ *Foundations of Physics* 24, 379 (1994). The origin of Boxworld can perhaps be traced to this paper.
21. John Bell’s remark that ‘correlations cry out for explanation’ is from his paper ‘Bertlmann’s socks and the nature of reality,’ originally published in *Journal de Physique*, Colloque C2, suppl. au numero 3, Tome 42, 4161 (1981), and reprinted in J. S. Bell, *Speakable and Un-speakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987), pp. 139–158. The quote (‘The scientific attitude is that correlations cry out for explanation.’) is on p. 55 in the original article, and on p. 152 in the collection.
22. ‘Yes! We Have No Bananas’ is a song from the 1922 Broadway revue *Make It Snappy*, by Frank Silver and Irving Cohn. Eddie Cantor sang the song in the revue, and it has been recorded by hundreds of artists since then. Apparently, there was a banana blight and a shortage of bananas at the time.
23. Bondi’s  $k$ -calculus: Hermann Bondi, *Relativity and Common Sense: A New Approach to Einstein* (Heinemann Educational Books, London, 1965), and *Assumption and Myth in Physical Theory* (Cambridge University Press, Cambridge, 1967).
24. The formulation of the light postulate as ‘no overtaking of light by light’ is from *Assumption and Myth in Physical Theory*, p. 27.
25. The formulation of the relativity principle as ‘velocity doesn’t matter’ is from *Relativity and Common Sense*, pp. 4, 58, 147.